



## EXTRATERRESTRIAL BIRTHDAYS

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### THE BIRTHDAY PROBLEM

- How many people must be in a room before the probability of at least two sharing a birthday is greater than 50%?




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### SOLUTION

- $P(\text{at least 2 share}) = 1 - P(\text{no one shares})$
- To find  $P(\text{no one shares})$ , do probabilities of choosing a non-matching birthday:

$$\begin{aligned} &= \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \cdots \times \left(1 - \frac{n-1}{365}\right) \\ &= \frac{365 \times 364 \times 363 \times \cdots \times (365 - (n-1))}{365^n} \\ &= \frac{365!}{(365-n)! \times 365^n} = \frac{{}_{365}P_n}{365^n} \end{aligned}$$

- So  $P(\text{at least 2 share}) = 1 - \frac{{}_{365}P_n}{365^n}$

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### STANDARD RESULT

$\bullet$  P(at least 2 share) =  $1 - \frac{365 P_n}{365^n}$

n	P(sharing)
5	2.71%
10	11.69%
15	25.29%
20	41.14%
25	56.87%
30	70.63%
35	81.44%
40	89.12%
45	94.10%
50	97.04%

$\bullet$  For  $n = 23$ , P(at least 2 share) = 50.73%

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### A SERIES APPROXIMATION

$\bullet$  P(no one shares) =  $\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \dots \times \frac{365-(n-1)}{365}$   
 $= \left(1 - \frac{0}{365}\right) \times \left(1 - \frac{1}{365}\right) \times \left(1 - \frac{2}{365}\right) \times \dots \times \left(1 - \frac{n-1}{365}\right)$

$\bullet$  Recall the Taylor Series:  $e^x = 1 + x + \frac{x^2}{2} + \dots + \frac{x^n}{n!} + \dots$

$\bullet$  So a first-order approximation is:  
 P(no one shares) =  $e^{-0/365} \times e^{-1/365} \times e^{-2/365} \times \dots \times e^{-(n-1)/365}$   
 $= 1 \times e^{-(1+2+\dots+(n-1))/365}$   
 $= e^{-(n(n-1)/2)/365}$

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### AN APPROXIMATE SOLUTION

Solving P(at least 2 share) > 0.50

$$1 - e^{-(n(n-1)/2)/365} > 0.5$$

$$e^{-(n(n-1)/2)/365} < 0.5$$

$$\frac{-n(n-1)}{2(365)} < \ln(0.5)$$

$$n(n-1) > (365) \ln 4 \approx 505.997$$

$$n^2 - n - 506 > 0$$

$$n > \frac{1 + \sqrt{1 + 4(506)}}{2} = \frac{1 + \sqrt{2025}}{2} = 23$$


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### THE MODE ANALYTICALLY

- We avoid calculus, since  ${}_dP_{x-1}$  is discrete.
- For which  $x$  is  $P(x) > P(x+1)$  ?

$$\frac{{}^{x-1}dP_{x-1}}{d^x} > \frac{{}^x dP_x}{d^{x+1}}$$

$$\frac{(x-1)}{d^x} \frac{d!}{(d-x+1)!} > \frac{x}{d^{x+1}} \frac{d!}{(d-x)!}$$

$$d(x-1) > x(d-x+1)$$

$$x^2 - x - d > 0$$

$$x > \frac{1 + \sqrt{1+4d}}{2} \quad \text{EXACT FORMULA!}$$

- For  $d = 365$ , we get  $x > 19.61$ , so 19 or 20.
- Approximating:  $x > \sqrt{d}$  for large  $d$

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### THE MEAN (EXPECTED VALUE)

- Since  $P(X=x) = \frac{{}^{x-1}dP_{x-1}}{d^x}$  for  $x \in \{2, 3, 4, \dots, d+1\}$   
then  $E(X) = \sum_{x=2}^{d+1} \frac{{}^{x-1}dP_{x-1}}{d^x}$

- and this is related to **Ramanujan's Q-function**, specifically  $E(X) = 1 + Q(d)$
- which is asymptotic:

$$E(X) \sim 1 + \sqrt{\frac{\pi d}{2}} - \frac{1}{3} + \frac{1}{12} \sqrt{\frac{\pi}{2d}} - \frac{4}{135d} + \dots$$



- and for  $d=365$ , we get:

$$E(X) \approx \sqrt{\frac{365\pi}{2}} + \frac{2}{3} + \frac{1}{12} \sqrt{\frac{\pi}{730}} - \frac{4}{49275} + \dots \approx 24.6166$$

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### BIRTHDAY PROBLEM ON JUPITER?

- A Jovian day = 9.9259 Earth-hours
- A Jovian year = 11.86231 Earth-years

$$1 \text{ Jyr} = 11.86231 \text{ Eyr} \left( \frac{365.24 \text{ Edays}}{1 \text{ Eyr}} \right) \left( \frac{24 \text{ Ehrs}}{1 \text{ Eday}} \right) \left( \frac{1 \text{ Jday}}{9.9259 \text{ Ehrs}} \right)$$

$$\approx 10476 \text{ Jdays}$$



$$P(X=x) = \frac{{}^{x-1}}{10476} ({}_{10476}P_{x-1}) \text{ for } x \in \{2, 3, 4, \dots, 10477\}$$

- Both  $10476^{25}$  and  ${}_{10476}P_{25}$  cause a TI-84 overflow!

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### JOVIAN BIRTHDAY AVERAGES

- Mean:

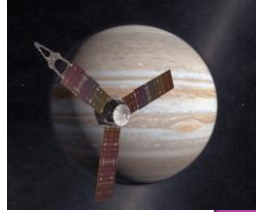
$$E(X) \approx \sqrt{\frac{\pi d}{2}} + \frac{2}{3} + \frac{1}{12} \sqrt{\frac{\pi}{2d}} - \frac{4}{135d} + \dots \approx 128.947$$

- Median:

$$n \approx \frac{1 + \sqrt{1 + 4d \ln 4}}{2} \approx 121.017$$

- Mode:

$$x \approx \frac{1 + \sqrt{1 + 4d}}{2} \approx 102.854$$




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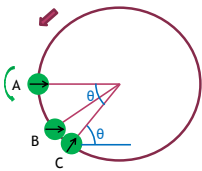
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### SIDERIAL VS. SOLAR TIME

- Prograde motion



- AB: 1 sidereal day

- Rotate  $2\pi$

- AC: 1 solar day

- Rotate  $(2\pi + \theta)$

- Revolve  $\theta$

- Circle: 1 orbital period

- Revolve  $2\pi$

- Equations:

$$\frac{T_{\text{sidereal}}}{T_{\text{solar}}} = \frac{2\pi}{2\pi + \theta}$$

$$\frac{T_{\text{solar}}}{T_{\text{orbital}}} = \frac{\theta}{2\pi}$$

- For retrograde motion: switch B & C, and

$$\frac{T_{\text{sidereal}}}{T_{\text{solar}}} = \frac{2\pi}{2\pi - \theta}$$

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### USING ASTRONOMICAL DATA

- Astronomical data typically includes the orbital period and the sidereal day length, but not the solar day length

- From the equations:  $\frac{T_{\text{sidereal}}}{T_{\text{solar}}} = \frac{2\pi}{2\pi \pm \theta}$ ,  $\frac{T_{\text{solar}}}{T_{\text{orbital}}} = \frac{\theta}{2\pi}$

we can substitute for  $\theta$  to get:  $T_{\text{solar}} = \frac{T_{\text{sidereal}}}{1 \mp \frac{T_{\text{sidereal}}}{T_{\text{orbital}}}}$

- Then: Days in a year =  $\frac{T_{\text{orbital}}}{T_{\text{solar}}} = \frac{T_{\text{orbital}}}{T_{\text{sidereal}}} \mp 1$

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## THE PLANETS

Planet	Siderial Day (Edays)	Orbital Period (Edays)	Days in a Year	Birthday Median
Mercury	58.646	87.9691	0.5000	1.47
Venus (retro)	243.019	224.701	1.9246	2.21
Earth	0.99727	365.24	365.24	23.01
Mars	1.025957	686.971	668.59	30.95
Jupiter	0.41354	4332.59	10475.8	121.01
Saturn	0.440417	10759.22	22428.6	184.53
Uranus (retro)	0.71833	30799.095	42877.0	244.30
Neptune	0.6713	60190.03	89660.9	353.06

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## THE DWARF PLANETS

Dwarf Planet	Siderial Day (Edays)	Orbital Period (Edays)	Days in a Year	Birthday Median
1 Ceres	0.3781	1680.99	4444.9	79.00
134340 Pluto	6.38723	90465.	14162.4	140.62
136108 Haumea	0.163146	103468.	634203.9	938.15
136472 Makemake	0.32379	113183.	349555.8	696.62
136199 Eris	1.079	204624.	189641.3	513.24

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## CLASSIFICATION

According to the IAU:

- ⊙ A planet must:
  1. Be in orbit around the sun
  2. Have sufficient mass to have reached hydrostatic equilibrium (a round shape)
  3. Have cleared the neighborhood around its orbit
- ⊙ If 1,2 only, then “dwarf planet”
- ⊙ If 1 only, but not a satellite (e.g. moon), then “small solar system body”




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## OTHER CELESTIAL BODIES

Small Solar System Body (SSSB)	Siderial Day (Edays)	Orbital Period (Edays)	Days in a Year	Birthday Median
2 Pallas (asteroid)	0.32555	1685.87	5177.5	85.22
2060 Chiron (centaur)	0.2466	18539.	75177.	323.3
1P/Halley (comet)	2.2	27500.	12499.	132.1
90377 Sedna (trans-Neptunian object, Oort cloud?)	0.42	4161000.	9907142.	3706.5

Planets do exist around other stars (exoplanets), and orbital period data is often available, but not siderial day lengths. [FUTURE RESEARCH]

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## THANK YOU!

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● A PDF of the presentation is available at:  
<http://www.milefoot.com/about/presentations/ETbirthdays.pdf>

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