Do Your Students Understand Logarithms?

Steven J. Wilson
KAMATYC, March, 2012

Mr. Wilson's
Our
Do ~~Your~~ Students Understand Logarithms?

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POP QUIZ !!!

- No discussions with your neighbor
- One minute time limit
- Complete the sentence:
  A logarithm is ____________________________.

Results in my Calculus Class: The Five Good Answers (of 36)

- The inverse of e^x
- The inverse of an exponential
- A function used to determine exponent
- The exponent required to produce a given #
- Inverse of Exponitioal [sic]

Results in my Calculus Class: The Six Basic Answers (of 36)

- A function
- A function
- A type of function
- A function
- Function
- A mathmatic function [sic]

Results in my Calculus Class: A Selection of the 25 Wrong Answers

- An expression to find unusual exponent rates
- A function that increases at a high rate
- Something I can use but can't define
- The derivative of an exponential
- Annoying
- The opposite of an exponent
- No idea but I think it has something to do with the number 10
- Base function (depending on specific base)
- One of the words for math. (I don't know)

Part 2 of the POP QUIZ!

Select ALL that apply:

A logarithm is:

a) A set of rules
b) An exponent
c) A number
d) An order of magnitude
e) A function
f) A transformation
g) An inverse

Part 2 Results from my Calculus Class

Answer	Percent Who Gave That Answer
Function	69%
Inverse	64%
Number	58%
Set of Rules	50%
Exponent	44%
Order of magnitude	42%
Transformation	39%

Nick Boredaki Speaks His Mind

(Math Horizons, November 2011)

Dear Nick:

I went to my first college math class (Calc II) and the professor on the first day says he's going to do natural logarithms the "right way." I'm thinking OK, we did logs in AP calc and I totally nailed them. Logs are just the opposites (I know, "inverses") of exponential functions. I could do growth and decay problems with my eyes closed!

Nick Boredaki Speaks His Mind

(Math Horizons, November 2011)

Now my loggerhead college prof tells us that we really don't know what an exponential function is and then defines the log to be an integral! What the F(unction)?!? I was good with *e* to the *x* – can't I just keep doing it that way?

Exponentially confused,

Lenny Lost in the Last Row

Parts of His Answer

(Math Horizons, November 2011)

- In High school I learned five words about logs: "A log is an exponent." Those five words changed my life and got me through all sorts of scuffles with logarithms …

- But … advanced math is all about precise and formal definitions. And my great five-word salvation is no definition.

Parts of His Answer

(Math Horizons, November 2011)

- Why do professors torture us by stressing this definition in calc II? One answer is that it shows how useful the integral is. I agree it's pretty cool, but I don't think most calc II students get it. The real answer is that math profs get off on defining things generally and precisely, even if the time is not right.

The College Algebra Course at JCCC

1. Functions
2. Polynomial and Rational Functions
3. Exponential and Logarithmic Functions
4. Systems (and maybe Matrices)
5. Introduction to Sequences and Series

The Exponential Chapter

1. Exponential Functions
2. Logarithmic Functions
3. Laws of Logarithms
4. Exponential and Logarithmic Equations
5. Modeling with Exponential and Logarithmic Functions

The Logarithm Chapter from a 1958 College Algebra Textbook

1. Definition
2. Useful Properties
3. Systems of Logs
4. Characteristic & Mantissa
5. Rules for Characteristics
6. Tables of Logs
7. Reading Log Tables
8. Interpolation
9. Proportional Parts Tables
10. Logarithmic Computations
11. Products & Quotients
12. Cologarithms
13. Positive Powers & Roots
14. Negative Powers
15. Negative Numbers
16. Sums & Differences
17. Bases Other Than 10
18. Exp & Log Equations
19. Graph of a Log Function
20. Graphing on Log Scales

How Much Logarithmic Material?

Book Section	Logs in Book: Focus (incidental)	Logs in My Notes: Focus (incidental)	Hours on Logs: Focus (incidental)
1. Exp Functions	0%	0%	0
2. Log Functions	100%	100%	1.25
3. Laws of Logs	100%	100%	1.25
4. Equations	38% (77%)	31% (88%)	0.39 (1.10)
5. Modeling	36% (73%)	0% (100%)	0 (1.25)
TOTALS	55% (70%)	46% (78%)	2.89 (4.85)

My Book's Questions about Log Functions

- Express $\log_5 25 = 2$ in exponential form
- Evaluate $\log_9 81$
- Use the definition to find *x*: $\log_4 x = 2$
- Graph $y = \log_2(x-4)$
- Find domain of $y = \log_2(x-4)$

What are we teaching?

Logs as inverses, numbers, exponents, functions

Logs as Exponents?

- A logarithm is an exponent!
- When evaluating logarithms, remember that *a logarithm is an exponent.*
- It is important to understand that $\log_a x$ is an *exponent*.
- A *logarithm* is merely a name for a certain exponent.
- *Remember:* A logarithm is an exponent.

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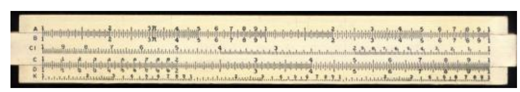
Logs as Numbers and Order of Magnitude

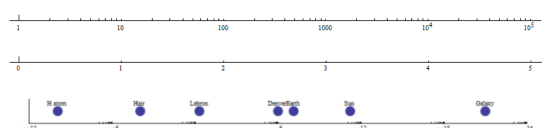
- Collect the following information from the internet. Use the same unit of length for each. Do NOT use scientific notation.
  - The diameter of a hydrogen atom
  - Thickness of a human hair
  - The height of Lebron James (plays basketball for the Miami Heat)
  - The distance from Kansas City to Denver
  - The diameter of the earth
  - The distance from the earth to the sun
  - The diameter of the Milky Way Galaxy
- Compute the logarithm of each number.
- Explain how the logarithms are growing.
- Why use logarithms rather than the original number?

Results

Item	Values	Logarithms
Hydrogen atom	0.000000000106 m	- 9.80
Human hair	0.0001 m	- 4.00
Lebron James	2.01 m	0.30
Kansas City to Denver	970900 m	5.99
Diameter of the earth	12756000 m	7.11
Earth to Sun	150000000000 m	11.18
Milky Way Galaxy	950000000000000000000 m	20.98

Log Scale on a Slide Rule





Logs as Functions

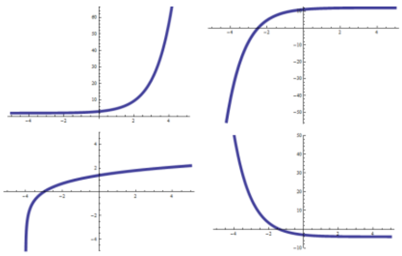
- For the function $f(x)=3\log(2x+7)-1$
- Find:
  - Domain and Range
  - Intercepts (both x and y)
  - Asymptote
  - Interval of increase or decrease

Survey of Current Books

Book	Domain/range	Intercepts	Asymptote	Increase/Decrease
#1	Domain only exercises 57-64	No	Yes exercises 57-64	No
#2	No examples, exercises 81-84	No examples, exercises 81-84	No examples, exercises 81-84	No
#3	Domain only exercises 39-50	Yes exercises 39-50	Yes exercises 39-50	Mentioned in text no exercises
#4	Yes exercises 53-62	No	Yes exercises 53-62	No
#5	Yes, exercises 45-56, 75-90	No	Yes, exercises 45-56, 75-90	No

Logs as Functions

Identify the log graph, then write its equation.



My Book's Questions about the Laws of Logarithms

- Expand $\log \frac{a^2}{b^4\sqrt{c}}$
- Combine $\ln 5 + 2\ln x - 3\ln\left(x^2+5\right)$

What are students learning from this section?
Logs as sets of rules?

My Book's Questions about Equations: Solve $2^{3x+1} = 3^{x-2}$.

The Typical Solution:

$$
\begin{aligned}
2^{3x+1} &= 3^{x-2}\\
(3x+1)\log 2 &= (x-2)\log 3\\
x(3\log 2 - \log 3) &= -\log 2 - 2\log 3\\
x &= \frac{-\log 2 - 2\log 3}{3\log 2 - \log 3}\\
x &\approx -2.947
\end{aligned}
$$

Why not continue?

$$
\begin{aligned}
x &= \frac{-\log 2 - 2\log 3}{3\log 2 - \log 3}\\
&= \frac{-(\log 2 + \log 9)}{\log 8 - \log 3}\\
&= \frac{-\log 18}{\log \frac{8}{3}}\\
&= \log_{8/3} \frac{1}{18}
\end{aligned}
$$

My Book's Questions about Equations: Solve $2^{3x+1} = 3^{x-2}$.

Which solution form is best?

- $x \approx -2.947$. Numerical approximation is fine. Why not use graphing? Are logs important?
- $x = \log_{8/3} \frac{1}{18}$. Exact solutions in simplified form. Students weary? Applications unimportant?
- $x = \frac{-\log 2 - 2\log 3}{3\log 2 - \log 3}$. For what purpose?

My Book's Questions about Equations: Solve $\log(x+2) + \log(x-1) = 1$.

$$
\begin{aligned}
\log(x+2) + \log(x-1) &= 1\\
\log(x+2)(x-1) &= 1\\
(x+2)(x-1) &= 10^1\\
x^2 + x - 2 &= 10\\
x^2 + x - 12 &= 0\\
(x+4)(x-3) &= 0
\end{aligned}
$$

The solutions appear to be $x = -4$ and $x = 3$
but $x = -4$ is extraneous, so $x = 3$.

So what was learned from this problem?

My Book's Presentation of Modeling with Logs: pH Levels

- A definition: $\mathrm{pH} = -\log\left[\mathrm{H}^+\right]$
- Interpretation: $\mathrm{pH} < 7$ is acidic, $\mathrm{pH} > 7$ is basic
- Table of common substances and their pH
- A notice: if pH increases by 1, $[H^+]$ decreases by a factor of 10
- Example: A sample of human blood had $\left[\mathrm{H}^+\right] = 3.16\times 10^{-8} M$
  Find the pH and classify it as acidic or basic.

  So what has the student learned? Chemistry?
  The log is a number?

My Book's Presentation of Modeling with Logs: Earthquake Intensity

- A definition: $M = \log \frac{I}{S}$
- Table of largest earthquakes and their magnitude
- A notice: "An earthquake of magnitude 6 is ten times stronger than an earthquake of magnitude 5."
- Example: The 1906 San Francisco earthquake measured 8.3. Another earthquake was 4 times as intense. What was its magnitude? [Ans. 8.9]

So what has the student learned? Geology?
The old way of measuring earthquakes?

My Book's Presentation of Modeling with Logs: Sound Intensity

- A definition: $B = 10 \log \frac{I}{I_0}$ measures psychological sensation of loudness.
- The reference intensity: $I_0 = 10^{-12}$ W/m^2
- Table of common sounds and their decibel levels
- Example: Find the decibel intensity level of a jet engine during takeoff if intensity was 100 W/m^2.

So what has the student learned? Psychology?

Understanding?

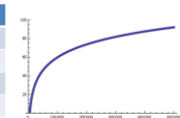
- Are we asking students to understand applications based on their [lack of] understanding of logs?
- Or are we asking students to understand logs based on their [lack of] understanding of applications?
- What applications can we expect students to understand?
  - Distance
  - Time
  - Money
  - Temperature

Growth of Money

- At 5% interest compounded continuously, how long will it take for \$5000 to grow to \$*B*?

$$A = Pe^{rt}$$
$$B = 5000e^{0.05t}$$
$$t = 20 \ln \frac{B}{5000}$$

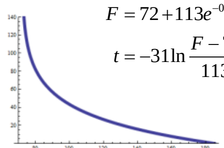
B	t (years)
\$5,000	0
\$10,000	13.8
\$15,000	21.9
\$20,000	27.7
\$40,000	41.0



Hot Coffee

- How long does it take a cup of 185° coffee to cool to F° Fahrenheit?

$$T = T_S + (T_0 - T_S)e^{-kt}$$
$$F = 72 + 113e^{-0.032t}$$
$$t = -31 \ln \frac{F-72}{113}$$



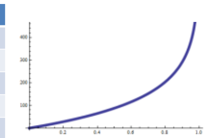
Temp (F)	Time (min)
185	0
140	15.7
120	26.5
100	43.3
80	82.1

Spent Fuel

- How long will spent fuel with a half-life of 87.7 years take to lose 100*p*% of its radioactivity?

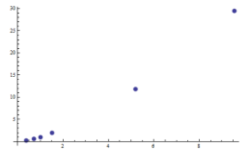
$$y = y_0 e^{-kt}$$
$$\frac{1}{2} y_0 = y_0 e^{-k(87.7)}$$
$$k = \frac{\ln 2}{87.7} \approx 0.0079$$
$$(1-p) y_0 = y_0 e^{-0.0079t}$$
$$t = -127 \ln(1-p)$$

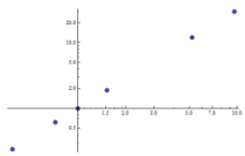
p	t (yrs)
50%	87.7
90%	291.3
99%	582.7
99.9%	874.0
99.99%	1165.3
99.999%	1456.7



Solar System: Distance and Period

Planet	Distance (AU)	Period (yrs)
Mercury	0.387	0.241
Venus	0.723	0.615
Earth	1.000	1.000
Mars	1.524	1.881
Jupiter	5.203	11.862
Saturn	9.555	29.458





$$\ln P = m \ln D$$

$$P = D^m$$

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Thank You!

- For copies of the slides, see:
  http://www.milefoot.com/about/presentations/UnderstandLogs.pdf
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