Dynamic Web Tools for Exploration, Modeling, & Assessment

– 15th Annual Kansas City Regional Mathematics Technology Expo –

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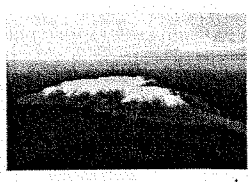
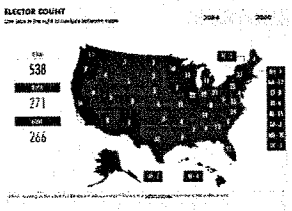
Today's Outline

- Steve Wilson
  - Social Choice / Trigonometry / Complex Analysis
- Mike Martin
  - Pharmacology / Cell Populations / Heart Dynamics
- History, Integration, & Issues
- web*Mathematica* Coding
- Invitation



Social Choice

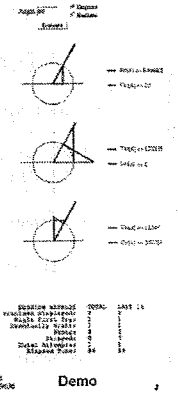
- Should we be concerned about the Electoral College in next week's election?
  - Important states?
  - Irrelevant states?
- Weighted Voting
  - Players & Quotas
  - Power Indices
- Implications
  - Motivation
  - Exploration
  - Availability
  - Non-technical



Demo

Trigonometry

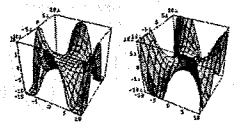
- Have I mastered the trig ratios?
  - All 6 ratios
  - Exact values
- Assessment
  - Immediate feedback
  - Detailed results
  - Printable for a grade
  - Accuracy
  - Speed

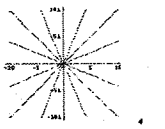


Demo

Complex Analysis

- How can we visualize a complex-valued function?
  - Calculus
  - ODE
- Views
  - 3D Surfaces
  - Contour plots
- Implications
  - Understand classes of functions
  - Visualize analytic results





Demo

Benefits of Dynamic Web Tools

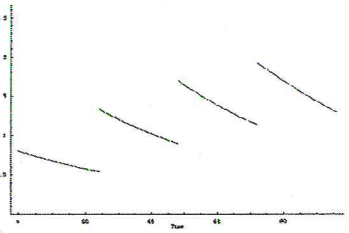
- Fosters synthesis
  - Numerical
  - Graphical
  - Analytical
- Fosters exploration
  - Parameter variation
  - Iterative processes
  - Higher-dimensional geometries
  - Analysis of models
- Fosters conceptual understanding
  - Implication of computations
  - Avoids technical issues
- Assessment
  - Practice to mastery
  - Immediate feedback
- Availability
  - To all from anywhere
  - Not just a campus lab

Drug Delivery:
Uniform Dosing with Exponential Decay

$$\frac{dc}{dt} = -\frac{c}{\tau}$$

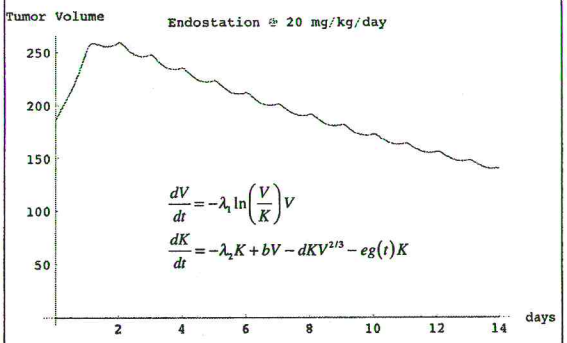
with a dosing of c_o
on each interval of length T
for a total of N intervals

$$c(t) = c_o e^{-t/\tau}$$



How can you interpret τ?
What does the value of τ mean relative to that of T?
Overcoming thresholds

Exploration



$$\frac{dV}{dt} = -\lambda_1 \ln\left(\frac{V}{K}\right) V$$

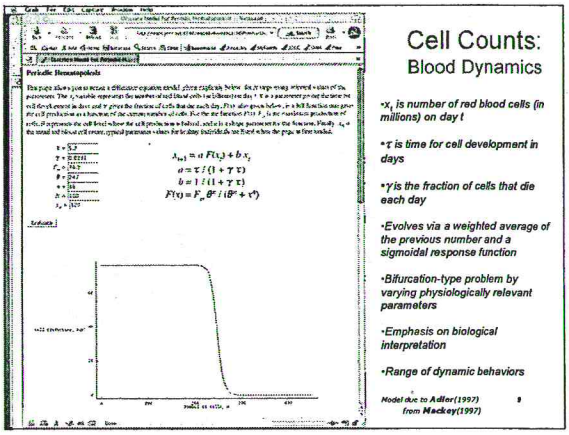
$$\frac{dK}{dt} = -\lambda_2 K + bV - dKV^{2/3} - eg(t)K$$

Progressive Complexity

- Exponential model: $x_{t+1} = r\, x_t \quad \Rightarrow \quad x(t) = x_o r^t$
- Logistic model: $x_{t+1} = r\,(1 - x_t)\, x_t$
  - With Harvesting: $x_{t+1} = r\,(1 - x_t)\, x_t - h\, x_t$
- Ricker model: $x_{t+1} = r\, e^{-x_t}\, x_t$
- Selection model: $p_{t+1} = \dfrac{s\, p_t}{s\, p_t + r\,(1 - p_t)}$

$$p_t = \frac{m_t}{m_t + x_t} \quad \text{with} \quad x_{t+1} = r\, x_t, \quad m_{t+1} = s\, m_t$$

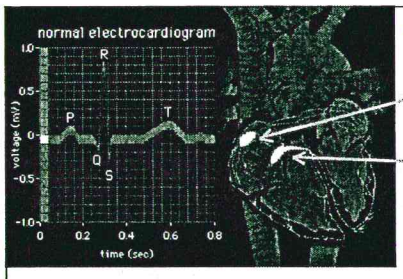
- Gas Exchange in Lungs: $c_{t+1} = (1-q)\, c_t + q\, \gamma$
- Sigmoidal PCRR: $x_{t+1} = r\, \dfrac{\theta^n}{\theta^n + x_t^n}\, x_t$



Cell Counts:
Blood Dynamics

- x_t *is number of red blood cells (in millions) on day t*
- τ *is time for cell development in days*
- γ *is the fraction of cells that die each day*
- *Evolves via a weighted average of the previous number and a sigmoidal response function*
- *Bifurcation-type problem by varying physiologically relevant parameters*
- *Emphasis on biological interpretation*
- *Range of dynamic behaviors*

Model due to ***Adler****(1997) from* ***Mackey****(1997)*

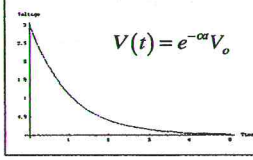


Heart Dynamics

Components

- SA Node
  - Clock or pacemaker
- AV Node
  - Response unit
- Heart
  - Blood pump

I-D Wave Propagation



$$V(t) = e^{-\alpha t} V_o$$

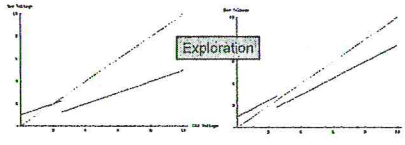
V_t is AV nodal potential after response to signal from the SA node
– potential decays exponentially during the time, τ, in between normal beats

Model due to ***Adler****(1997) from* ***Keener****(1981, 1998)*

AV Nodal Dynamics

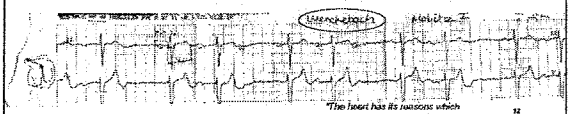
- To beat or not – depends on AV response
  - If potential is too high, heart has not recovered from last beat and ignores signal;
  - otherwise, node accepts signal, tells heart to beat, and increases its potential by fixed amount, call it u

$$V_{t+1} = \begin{cases} e^{-\alpha\, \tau} V_t & \text{if} \quad e^{-\alpha\, \tau} V_t > V_c \\ e^{-\alpha\, \tau} V_t + u & \text{if} \quad e^{-\alpha\, \tau} V_t \le V_c \end{cases}$$



In Vivo – In Situ

Video courtesy of Mark Fishman, Harvard University



"The heart has its reasons which reason does not know."
– Blaise Pascal

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History, Integration, & Issues

- What is web*Mathematica*?
  - Uses web server technology & *Mathematica* engine
  - Runs through a web browser or other web clients
- Timeline & Involvement
  - Wolfram & JCCC
  - Traditional, Online & Hybrid Classes (WebCT)
  - Interdisciplinary & "Client" Disciplines
- Amateur License Issues
  - Can use "the power of a specific capability of *Mathematica*"
  - No "arbitrary, open-ended calculation requests"
  - Page must be publicly accessible
  - Banner link is required

POWERED BY webMATHEMATICA

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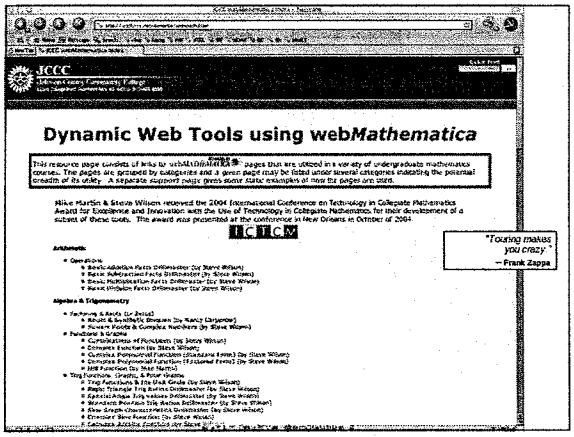

Coding Rudiments

```
<%@ page language="java" %>
<%@ taglib uri="/webmathematica-taglib" prefix="msp" %>
<html><head></head><body>
<msp:allocateKernel>
```

On the Server

```
<form action="page.jsp" method="post">
Enter a parameter value for
f(x)=x<sup>2</sup> + a y<sup>2</sup><br>

<input type="text" name="parameter">
<input type="submit" name="button" value="Evaluate"><br>
 <msp:evaluate>  MSPBlock[{$$parameter}, MSPShow[
Plot3D[x^2+ $$parameter *y^2,{x,-5,5},{y,-5,5}] ]  ]
 </msp:evaluate>

 </form>   </msp:allocateKernel>  </body></html>
```

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References

- Adler, F. *Modeling the Dynamics of Life: Calculus and Probability for the Life Sciences.* Brooks/Cole, 1997
- *BIO 2010: Transforming Undergraduate Education for Future Research Biologists.* National Research Council of the National Academies, 2003
- *CRAFTY Curriculum Foundations Report: Voices from the Partner Disciplines.* MAA, 2004
- Guyton, A. C. *Textbook of Medical Physiology* (8th Ed). Saunders, Philadelphia, 1991
- Keener, J. P., and J. Sneyd. *Mathematical Physiology,* Springer-Verlag, 1998
- Keener, J. P. On Cardiac Arrhythmias: AV Conduction Block. *Journal of Mathematical Biology,* 12:215-225, 1981
- Mackey, M. *Case Studies in Mathematical Modeling: Ecology, Physiology, & Cell Biology.* Prentice Hall, 1997
- Mathews, J.H., R.W. Howell. *Complex Analysis for Mathematics & Engineering.* Jones & Bartlett, 1997
- Tannenbaum, P. *Excursions in Modern Mathematics* (5th Ed). Prentice Hall, 2003
- *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004.* MAA, 2004

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