

## Dynamic Web Tools for Trigonometry,

version 2.0

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UMATYC  
St. George, 2010

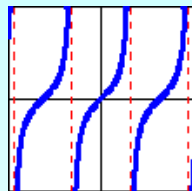
## Before the Web

- ◆ 1969: ARPANET
- ◆ 1974: TCP/IP (internet protocol)
- ◆ 1977: Commodore PET, Apple II
- ◆ 1981: MS-DOS
- ◆ 1982: Timex Sinclair 1000 (under \$100)
- ◆ 1985: Microsoft *Excel* appears
- ◆ 1988: *Mathematica* version 1
- ◆ 1990: first description of HTML (for web pages)

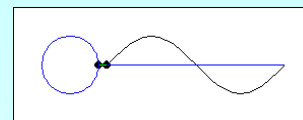


## The Still Page

- ◆ No user input
  - Fixed information
  - Static page
- ◆ Not dynamic



## The Animated GIF



- ◆ No user input
  - Fixed information
  - Motion on page
- ◆ Not dynamic

## Dynamic ???

- ◆ Dictionary definition
  - relating to energy or physical force in motion
  - energetic; vigorous; forceful
- ◆ In our context
  - activity – of user

## Baby Web

- ◆ 1990: first description of HTML (for web pages)
- ◆ 1990: Windows 3.0 (first successful Windows)
- ◆ 1991: WorldWideWeb (first browser)
- ◆ 1993: Mosaic browser (first with inline images)
- ◆ 1993: CGI scripts (first custom web pages)
- ◆ 1995: JavaScript (first client-side script)
- ◆ 1999: "Web 2.0" coined (as platform, interactive)
- ◆ 2001: web*Mathematica* 1.0 appears



## Excel and webMathematica

Special Angle Sine and Cosine Values  
by Steven J. Wilson

This page generates electronic flash cards so you measure. Enter your **exact fractional** answers as in  $\text{Sqrt}[3]$ .

$\cos\left(\frac{3\pi}{4}\right) =$

Check it New Problem

Date and Time: 2007/4/7, 19:8:41.820445  
Session ID: 66951377.20070407190841.697.

POWERED BY webMATHEMATICA The Dynamic Web

## Types of Tools

- ◆ Drill and Practice
- ◆ Conceptual Understanding
- ◆ Variation of Parameters

## Crossroads in Mathematics, 1995

- ◆ "In fact, the use of technology, coupled with a decreased emphasis in some traditional content areas, should provide the time that is needed to implement the needed reforms in mathematics education."

- discussion of Standard P-1, p. 16

## Drillmaster Examples

- ◆ Right Triangle Trig Ratios
- ◆ Standard Position Trig Ratios
- ◆ Special Angle Trig Values
- ◆ Special Angle Sine and Cosine Values

Right Triangle Trig Ratios Drillmaster  
by Steven J. Wilson

This page generates electronic flash cards so you can practice determining the basic trig ratios for right triangles. Enter (no decimals) in the input boxes. For square roots, use the function Sqrt with brackets (not parentheses), as in  $\text{Sqrt}[3]$ .

$\sin B = \frac{7}{\text{Sqrt}[149]}$  Correct  
 $\cos D = \frac{10}{\text{Sqrt}[149]}$  Correct  
 $\tan B = \frac{7}{10}$  Correct  
 $\cot B = \frac{10}{7}$  Correct  
 $\sec D = \frac{\text{Sqrt}[149]}{10}$  Correct  
 $\csc B = \frac{\text{Sqrt}[149]}{7}$  Correct

SESSION RESULTS	TOTAL	LAST 10
Problems Displayed:	2	2
Right First Try:	2	2
Eventually Right:	2	2
Wrong:	0	0
Skipped:	0	0
Total Attempts:	2	2
Elapsed Time:	201	201

Check it New Problem

Date and Time: 2006/9/7, 13:14:57.028955  
Session ID: 4414150600.20060907131136.1057

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Standard Position Trig Ratios Drillmaster  
by Steven J. Wilson

This page generates electronic flash cards so you can practice determining the basic trig ratios for points on the terminal position. Enter your **exact fractional** answers (no decimals) in the input boxes. For square roots, use the function Sqrt as in  $\text{Sqrt}[3]$ . If an answer is undefined, leave the input box blank.

Point  $(0, 7)$  is on the terminal side of  $\theta$  in standard position.

$\sin \theta = \frac{7}{\text{Sqrt}[130]}$  Correct  
 $\cos \theta = \frac{0}{\text{Sqrt}[130]}$  Wrong  
 $\tan \theta = \frac{0}{\text{Sqrt}[130]}$  Wrong  
 $\cot \theta = \frac{9}{7}$  Correct  
 $\sec \theta =$  Wrong  
 $\csc \theta =$  Wrong

SESSION RESULTS	TOTAL	LAST 10
Problems Displayed:	2	2
Right First Try:	0	0
Eventually Right:	0	0
Wrong:	1	1
Skipped:	1	1
Total Attempts:	2	2
Elapsed Time:	173	173

Check it New Problem

Date and Time: 2006/9/7, 13:8:51.687065  
Session ID: 3074194447.20060907130359.1446

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### Special Angle Trig Values Drillmaster


by [Steven J. Wilson](#)

This page generates electronic flash cards so you can practice determining the trig values for the basic special angle measure. Enter your **exact fractional** answers (no decimals) in the input boxes. For square roots, use the function as in Sqrt[3]. If an answer is undefined, leave the input box blank.

$\sin\left(\frac{\pi}{4}\right) = \text{Sqrt}[2]/2$  Correct  
 $\cos\left(\frac{\pi}{4}\right) = 1/\text{Sqrt}[2]$  Correct  
 $\tan\left(\frac{\pi}{4}\right) = 1$  Correct  
 $\cot\left(\frac{\pi}{4}\right) = 1$  Correct  
 $\sec\left(\frac{\pi}{4}\right) = \text{Sqrt}[2]$  Correct  
 $\csc\left(\frac{\pi}{4}\right) = \text{Sqrt}[2]$  Correct

SESSION RESULTS	TOTAL	LAST 10
Problems Displayed:	1	1
Right First Try:	1	1
Eventually Right:	1	1
Wrong:	0	0
Skipped:	0	0
Total Attempts:	1	1
Elapsed Time:	86	86

Date and Time: 2006/9/7, 13:21:53 863551  
 Session ID: 9275604832.20060907131938.776

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### Special Angle Sine and Cosine Values Drillmaster


by [Steven J. Wilson](#)

This page generates electronic flash cards so you can practice determining the sines and cosines of the basic special angle measure. Enter your **exact fractional** answers (no decimals) in the input boxes. For square roots, use the function as in Sqrt[3].

$\sin(210^\circ) = -1/2$  Correct

SESSION RESULTS	TOTAL	LAST 10
Problems Displayed:	2	2
Right First Try:	2	2
Eventually Right:	2	2
Wrong:	0	0
Skipped:	0	0
Total Attempts:	2	2
Elapsed Time:	15	15

Date and Time: 2007/5/9, 17:42:40 333913  
 Session ID: 9648703612.20070509174225.1041


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## Drillmaster Features

- ◆ Can use exact arithmetic
- ◆ Immediate feedback
- ◆ Printable for a grade (or screen capture)
- ◆ Detailed results
- ◆ Measure accuracy and speed

## More Drillmaster Examples

- ◆ Converting Radians to Degrees
- ◆ Converting Degrees to Radians
- ◆ Sine Graph Characteristics



### Converting Radians to Degrees Drillmaster


by [Steven J. Wilson](#)

This page generates electronic flash cards so you can practice converting the basic radian measure and answer in **simplest form** in the input box. Use digits only, with no decimal points.

$\frac{4\pi}{3} = 240$  ° Correct

SESSION RESULTS	TOTAL	LAST 10
Problems Displayed:	10	10
Right First Try:	10	10
Eventually Right:	10	10
Wrong:	0	0
Skipped:	0	0
Total Attempts:	10	10
Elapsed Time:	52	52

Date and Time: 2006/9/7, 13:18:1 585689  
 Session ID: 1602973834.2006090713179.69853

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### Converting Degrees to Radians Drillmaster


by [Steven J. Wilson](#)

This page generates electronic flash cards so you can practice converting the basic degree measure and answer in **simplest form** in the input box. For  $\pi$ , use Pi.

$330^\circ = 11\pi/6$  Correct

SESSION RESULTS	TOTAL	LAST 10
Problems Displayed:	6	6
Right First Try:	5	5
Eventually Right:	6	6
Wrong:	0	0
Skipped:	0	0
Total Attempts:	7	7
Elapsed Time:	39	39

Date and Time: 2007/5/9, 17:45:45 494200  
 Session ID: 7285966126.2007050917455.87619

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**Sine Graph Characteristics Drillmaster**  
by Steven J. Wilson

This page generates electronic flash cards so you can practice determining the basic characteristics of a sine function. E answers (no decimals) in the input boxes. For  $\pi$ , use Pi. (Phase shift is not requested, since that answer depends on both cosine) to describe the graph, and the sign of the coefficient of the function.

$f(x) = 2 + 4 \sin(9x)$

Period =  Correct

Amplitude =  Correct

Vertical Shift =  Correct

SESSION RESULTS		TOTAL	LAST 10
Problems Displayed:	1	1	1
Right First Try:	1	1	1
Eventually Right:	1	1	1
Wrong:	0	0	0
Skipped:	0	0	0
Total Attempts:	1	1	1
Elapsed Time:	30	30	

Date and Time: 2006/07, 13:23:22 145492  
Problem ID: 2012041943-20160571322413272

## CUPM Curriculum Guide, 2004

“Of course faculty in other disciplines want students to possess the computational skills required for their subjects. But they especially want students to possess conceptual understanding ... experience with mathematical modelling ... and to have the communication skills ...”

- discussion of Recommendation 1, p. 12

## CUPM Curriculum Guide, 2004

“At every level of the curriculum, some courses should incorporate activities that will help all students progress in learning to use technology

- ◆ Appropriately and effectively as a tool for solving problems;
- ◆ As an aid to understanding mathematical ideas.”

- Recommendation 5, p. 22


## Beyond Crossroads, 2006

“Technology can be used by mathematics educators to enhance conceptual understanding through a comparison of verbal, numerical, symbolic, and graphical representations of the same problem.”

- Chap. 7, Teaching with Technology, p. 56

## Conceptual Understanding Examples

- ◆ Trig Values and the Unit Circle
- ◆ Exploring Fourier Series
- ◆ 2D Matrix Transformations
- ◆ Complex Exponential Function



**Trig Values and the Unit Circle**  
by Steven J. Wilson

You can use this page to investigate values of the trigonometric functions and their relationship to the unit circle. For n

Angle:   Degrees  Radians

—  $\sin(x) = 0.866025$ 
—  $\tan(x) = -1.73205$ 
—  $\csc(x) = 1.1547$   
—  $\cos(x) = -0.5$ 
—  $\sec(x) = -2.$ 
—  $\cot(x) = -0.57735$

**Exploring Fourier Series**  
by Steven J. Wilson

You can use this page to investigate the behavior of functions defined in the manner of Fourier series. These are given by the formula

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

All functions derived in this manner will be periodic. Some very interesting graphs are possible with simple formulas for  $a_n$  and  $b_n$ .

Coarse Coefficient Sequence,  $a_n = \frac{(-1)^n}{(n^2)}$

Fine Coefficient Sequence,  $b_n = \frac{1}{n}$

Constant Term,  $a_0/2 = 0$

Sequence Type:  
 All terms  
 Even terms only  
 Odd terms only

Terms (max 50),  $n = 20$

$$y = -\cos(x) + \frac{1}{4} \cos(2x) - \frac{1}{9} \cos(3x) + \frac{1}{16} \cos(4x) - \frac{1}{25} \cos(5x) + \frac{1}{36} \cos(6x) - \frac{1}{49} \cos(7x) + \frac{1}{64} \cos(8x) - \frac{1}{81} \cos(9x) + \frac{1}{100} \cos(10x) - \frac{1}{121} \cos(11x) + \frac{1}{144} \cos(12x) - \frac{1}{169} \cos(13x) + \frac{1}{196} \cos(14x) - \frac{1}{225} \cos(15x) + \frac{1}{256} \cos(16x) - \frac{1}{289} \cos(17x) + \frac{1}{324} \cos(18x) - \frac{1}{361} \cos(19x) + \frac{1}{400} \cos(20x)$$

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Automatic Updates: 8/1/2004  
Technical Math  
Calculus  
Differential Equations  
2004 ICTM

**2D Matrix Transformations**  
by Steven J. Wilson

This program will enable you to analyze the geometric effects of a 2D transformation matrix in a Cartesian coordinate system. Enter the values of the matrix entries (any functions can be entered, using radians, as Cos(0.1) and Sin(0.1)), and the number of iterations to be drawn. Then press the Evaluate button.

Number of iterations?

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**The Complex Exponential Function**  
by Steven J. Wilson

By varying the parameters, you can use this page to study the behavior and some of the complex regions

$$z^2 = x^2 - y^2 + 2xyi$$

Note: Due to both algebraic and coordinate issues, some of the graphs may not be completely perfect.

Parameter Input (the imaginary values use a capital 'I')

Three graphs are shown (along the real and/or imaginary axis) of the 2D graphs of the complex exponential function.

The Dynamic Web Tools site

**CUPM Curriculum Guide, 2004**

- ◆ "Students should ... see the graphical effects of varying parameters."
- ◆ "Students can be encouraged to ask 'what if?' questions, ... Specific examples include studying the effects of manipulating parameters on classes of functions ..."

- discussion of Recommendation 5, p. 24

**Exploring Parameters:  
Examples using Excel**

- ◆ Exploring the Graph of the Sine Function
- ◆ Curvilinear Coordinates and the Sine Function
- ◆ Curvilinear Coordinates and the Tangent Function

**Exploring the Graph of the Sine Function**  
by Steven J. Wilson

$f(x) = -2.0 \sin( 2.0 x + 3.14 ) + 3.0$

**Curvilinear Coordinates and the Sine Function**  
by Steven J. Wilson

Coordinate System:

$f(t) = 3.2 \sin(4.7 t + 2.53) + 5.8$

t: Min: -11.56 Max: 9.61

**Curvilinear Coordinates and the Tangent Function**  
by Steven J. Wilson

Coordinate System:

$f(x) = 2.0 \tan(0.92 x + 0.00) + 2.0$

t: Min: -39.27 Max: 39.27

A maximum of 23 branches of the tangent graph can be shown.

### Creation of an *Excel* tool

- ◆ Use formulas to create the data table
- ◆ Add the graph, generated from the table
- ◆ Attach controls to appropriate table elements

### Characteristics of an *Excel* tool

- ◆ Uses decimal approximations
- ◆ User must have MS *Excel*
- ◆ Immediate response with an ActiveX slider (delayed with Control slider)
- ◆ Macro Security Level  $\leq$  Medium if ActiveX is used

### Exploring Parameters: Examples using *webMathematica 2*

- ◆ Rose Analyzer
- ◆ Limacon Analyzer
- ◆ Lissajous Curve Analyzer

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**Rose Analyzer**  
by Steven J. Wilson

This program will enable you to analyze the mathematical rose. Enter the parameters of the polar function.

$r(\theta) = a \sin(a\theta)$

and the maximum value of  $\theta$  (the minimum value of  $\theta$  will be zero).  
The value of  $\pi$  can be entered as Pi. Then press the Evaluate button.

Value of the parameter:

Maximum value of  $\theta$ :

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**Limacon Analyzer**  
by [Steven J. Wilson](#)

This program will enable you to analyze the limaçon. Enter the parameters  $a$  and  $b$  of the polar function:

$$r(\theta) = b + a \cos(\theta)$$

and the maximum value of  $\theta$  (the minimum value of  $\theta$  will be zero). The value of  $\pi$  can be entered as Pi. Then press the Evaluate button.

Value of the parameters:  
 $a =$    
 $b =$    
 Maximum value of  $\theta$ :

webMATHematica The Dynamic Web Tools site is managed by Mike Martin & Steve Wilson, with assistance from JCCC Computing Services

**Lissajous Curve Analyzer**  
by [Steven J. Wilson](#)

This program will enable you to analyze the family of curves of Lissajous. Enter the parameters of the parametric functions:

$$x(t) = a_1 \sin(bt_1 + c_1) + d_1$$

$$y(t) = a_2 \sin(bt_2 + c_2) + d_2$$

and the maximum value of  $\theta$  (the minimum value of  $\theta$  will be zero). The value of  $\pi$  can be entered as Pi. Then press the Evaluate button.

Value of the parameters:  
 $a_1 =$    
 $b_1 =$    
 $c_1 =$    
 $d_1 =$    
 $a_2 =$    
 $b_2 =$    
 $c_2 =$    
 $d_2 =$    
 Maximum value of  $\theta$ :

- But in 2007**
- ◆ MS *Office* was being shipped with higher default security settings
  - ◆ MS *Excel* no longer offered ActiveX controls
  - ◆ *Mathematica* introduced the Manipulate command
  - ◆ Two years later, a web version of the Manipulate command was available

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**Trig Values**  
by [Steven J. Wilson](#)

Move the slider to investigate values of the trigonometric functions and their relationship to the unit circle.

Angle:

$\sin[x] = 0.71934$      $\tan[x] = -1.03553$      $\csc[x] = 1.39116$   
 $\cos[x] = -0.694659$      $\sec[x] = -1.43956$      $\cot[x] = -0.945689$

134 Degrees  
2.33274 Radians

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- Creation of a web*Mathematica* tool**
- |   |   |
|---|---|
| <p>Version 1 (2001)</p> <ul style="list-style-type: none"> <li>◆ Write <i>Mathematica</i> code</li> <li>◆ Embed into HTML using MSP tags</li> <li>◆ Add HTML form commands for interaction</li> </ul> | <p>Version 3 (2009)</p> <ul style="list-style-type: none"> <li>◆ Write <i>Mathematica</i> code (Flash objects are generated)</li> <li>◆ Embed into HTML using JSP tags</li> </ul> |
|---|---|

- Characteristics of a web*Mathematica* tool**
- ◆ User requires only a web browser
  - ◆ Interaction via HTML form components or Flash technology
  - ◆ Can use exact or approximate arithmetic
  - ◆ *Mathematica*, web*Mathematica*, and a Java servlet container are on server

## Conclusions – for the instructor

- ◆ higher standards for drill and practice with less time
- ◆ foster conceptual understanding by relating numerical, graphical and symbolic representations
- ◆ real-time variation of parameters is visually very effective

## Web Pages

- ◆ Topics in Trigonometry
  - [staff.jccc.edu/swilson/trig/index.htm](http://staff.jccc.edu/swilson/trig/index.htm)
- ◆ Dynamic Web Tools using webMathematica
  - [staff.jccc.edu/mmartin/webmath.html](http://staff.jccc.edu/mmartin/webmath.html)
- ◆ Steven J. Wilson
  - [staff.jccc.edu/swilson/index.htm](http://staff.jccc.edu/swilson/index.htm)