

Math and Physics for Video Game Developers

Steven J. Wilson, Mathematics
J. Douglas Patterson, Astronomy
Johnson County Community College

KC Math Tech Expo, 2007

Current Market

- Video game hardware and software sales topped **\$12.5 billion** in 2006, **up 19%**
- Hardware sales **increased 43%** from 2005 to 2006
- The 2007 Game Developer's Conference in San Francisco was attended by **25,000 people**

JCCC Interest

- Growth of Video Game market suggested a need for programmers
- Computer Science division was experiencing a shift in the programming job market

Initial Institutional Planning

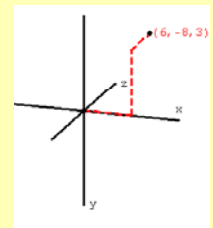
- An Associate Degree program
- College algebra prerequisite
- No physics prerequisite
- C++ computer prerequisite
- 4 credit hours

Our planning

- 4 credit hours would be 3 lecture, 2 lab
- Lectures: primarily math and physics
- Lab: implementing lecture material in C++
- Graded material: weekly homework & labs, 4 unit tests, final exam


Basic 2D & 3D Math

- Basic coordinates
- Distance Formula
- Lines, circles, spheres
- Bounding boxes
- Domains of basic functions



Basic Motion

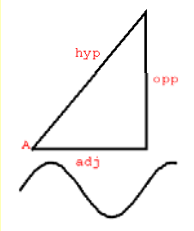
- Units of measurement
- Position, velocity, acceleration in 1D
- 4 equations of motion (cst acceleration)
- Free Fall
- Forward Euler method



www.brainybetty.com 7

Basic Trigonometry

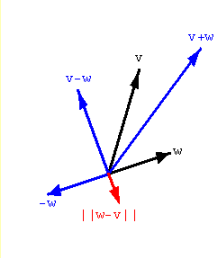
- Angle measures: degrees & radians
- Definitions & Graphs: sine, cosine
- Inverse functions: definitions, domain
- Identities: symmetric, translational
- Approximations: Taylor vs. Minimax



www.brainybetty.com 8

Basic Vectors

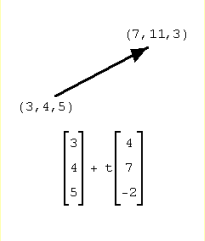
- Coordinate form
- Magnitude, heading and pitch
- Opposites, addition, subtraction
- Scaling, normalizing



www.brainybetty.com 9

Parametric Forms


- Line, Ray: defined by point and direction
- Convert to/from algebraic form
- Given point & direction, find new point
- Given 2 points, find direction



www.brainybetty.com 10

Projectile Motion


- Motion in 2D (generalizable to 3D)
- Vertical & horizontal components
- Acceleration due to gravity



www.brainybetty.com 11

Newton's Laws

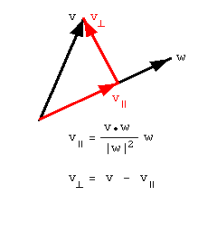
- Forces and accelerations
- Free body diagrams
- Normal forces
- Flat surfaces and Inclined planes
- Friction
- Wind resistance



www.brainybetty.com 12

Vector Dot Products

- Definition & properties
- Dot product equations: 2D line, 3D plane
- Angle between two vectors
- Vector projection
- Perpendicular & parallel components



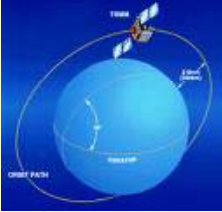
$$v_{||} = \frac{v \cdot w}{|w|^2} w$$

$$v_{\perp} = v - v_{||}$$

www.brainybetty.com 13

Circular Motion


- Centripetal forces
- Speed limits and tightest turns
- Gravity between two objects
- Circular orbits



www.brainybetty.com 14

Closest Points

- Closest point on a 3D line to a point
- Closest point on a plane to a point



www.brainybetty.com 15

Basic Matrices

- Definitions
- Equality of matrices
- Operations: add, subtract, scalar multiplication, transposition
- Determinants

$$A = \begin{bmatrix} 32 & -4 \\ 6 & 27 \end{bmatrix}$$

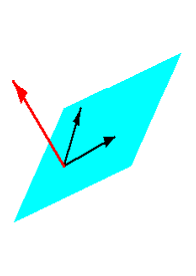
$$B = \begin{bmatrix} -2 & 17 \\ 5 & 76 \end{bmatrix}$$

$$2A + 3B = ?$$

www.brainybetty.com 16

Vector Cross Products

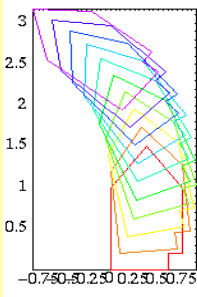
- Definition (as a determinant)
- Properties
- Finding equation of plane from 3 points



www.brainybetty.com 17

Matrix Multiplication

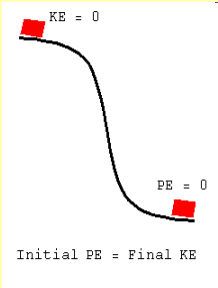
- Definition
- Identity matrices
- Homogeneous coordinates
- Matrices for scaling, translation, rotation



[web.Mathematics.demo](http://www.Mathematics.demo)
www.brainybetty.com 18

Work, Energy, and Power


- Work is a change in energy
- Kinetic energy: energy of motion
- Potential energy: stored energy
- Power: time rate at which work is done



www.brainybetty.com

Conservative Forces

- Conservative forces do no work (conservation of energy)
- Examples: gravity
- Using energy to find speed at bottom of a hill (assuming no friction)



www.brainybetty.com

Non-Conservative Forces


- Non-conservative forces can do work
- Examples: friction, air resistance, thrust, rope tension
- Using energy to find frictional force
- Finding speed at which work is done



www.brainybetty.com

Orbital Energy


- Gravitational Potential Energy
- Several sources of gravity
- Force fields



www.brainybetty.com

Collisions

- Momentum: $p = mv$
- Impulse: force in small time which changes momentum
- Conservation of momentum
- Recoil velocity
- Elastic and Inelastic collisions
- Reflections off walls



www.brainybetty.com

Rotation Matrices

- 2D rotations about origin
- 3D rotations about origin and axes
- Rotations about other points
- Rotations about another axis
- Concatenate all rotation matrices

$$\begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

www.brainybetty.com

Rotational Motion

- angular displacement, angular velocity, angular acceleration
- tangential and centripetal acceleration
- Moment of inertia (rotational analog of mass)
- Torque (rotational analog of force)



www.brainbuddy.com

25

The Labs

1. Collision detection (algebraically)
2. Linear accelerated motion (Forward Euler)
3. Force and motion (simulations of sliding box and a rocket)

www.brainbuddy.com

26

The Labs

4. Angles (vector class, unary functions)
5. Projections (class binary functions)
6. Closest Points (class ternary functions)
7. Scaling & Translations (adding matrix multiplication to the class)

www.brainbuddy.com

27

The Labs

8. Work and Energy (simulation of cart on a ramp and spacecraft around the earth)
9. Collisions and Momentum (object bouncing off of a plane, 2 objects colliding)
10. Rotations (about any axis and center)

www.brainbuddy.com

28

```
// m0202.cpp... Forward Euler method with given variable acceleration
// Steve Wilson, Jan. 31, 2007, home, MS Visual C++ 2001
// also tested in CLB 233, MS Visual C++ 2003

#include "stdafx.h"
#include <iostream>
#include <fstream>
using namespace std;

int main()
{
    // Declare variables
    const int arraySize = 1001; // array size
    double deltaT; // time step
    double x[arraySize]; // position
    double v[arraySize]; // velocity
    double a[arraySize]; // acceleration
    double t[arraySize]; // time
    int i = 0; // loop counter
    int totalSteps; // total number steps needed

    // Initialize variables
    t[0] = 0;
    cout << "Enter the initial position: ";
    cin >> x[0];
    cout << "Enter the initial velocity: ";
    cin >> v[0];
    a[0] = -9.8 - 0.1 * v[0]; // acc with wind resistance
    cout << "Enter the time step: ";
    cin >> deltaT;
```

```
// Print header to screen
cout << "Time \t Pos \t Vel \t Accel" << endl;
cout << 0 << "\t" << x[0] << "\t" << v[0] << "\t" << a[0] << endl;

// Compute and print results to screen
totalSteps = static_cast<int>( 10 / deltaT );
do {
    t[i + 1] = t[i] + (i + 1) * deltaT;
    a[i + 1] = -9.8 - 0.10 * v[i];
    v[i + 1] = v[i] + a[i + 1] * deltaT;
    x[i + 1] = x[i] + v[i + 1] * deltaT;
    cout << t[i + 1] << "\t" << x[i + 1] << "\t";
    cout << v[i + 1] << "\t" << a[i + 1] << endl;
} while (i++ < totalSteps);

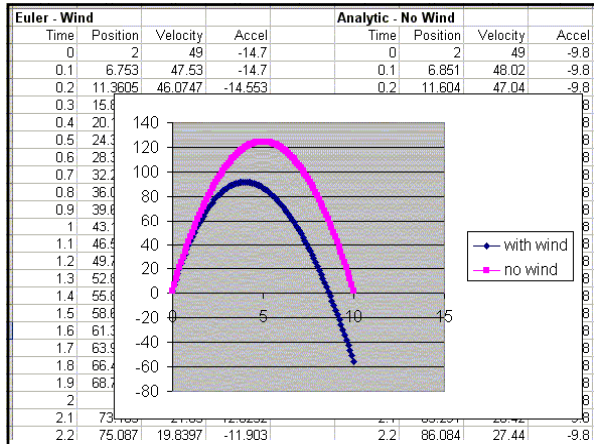

cout << endl;
cout << endl;

// Open sequential-access file
ofstream outMotion(
    "C:/Documents and Settings/All Users/Desktop/wind.dat",
    ios::out );

// write results to file
for (i = 0; i <= totalSteps; i++)
    outMotion << t[i] << "\t" << x[i] << "\t" << v[i] << "\t" << a[i] << "\n";

return 0;
// ofstream destructor will close file
}
```


10

Students

- 8 students in Spring 2007
- 6 were Game degree candidates
- All had trig, 5 had some calculus
- C++ experience was varied
- Almost perfect attendance
- Grades: A, A, B, C, C, D, D, W
- Feedback: useful & challenging class

www.brainybety.com 32



A Second Course ?

- Calculus prerequisite?
- Higher C++ prerequisite?
- Projections and viewing transformations
- Splines and Bezier Curves
- Quaternions
- Representing surfaces
- Lighting, shading, texturing

www.brainybety.com 33