Spent Fuel and Circuit Gain: What's in a Log?

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Disclaimer

- This presentation is **NOT** about
  - doing logs
  - rules of logs
  - computing logs
  - calculations with logs
  - etc. and so forth…
- Rather, it is about
  - understanding logs conceptually …

POP QUIZ !!!

- No discussions with your neighbor
- One minute time limit

- Complete the sentence:
  A logarithm is _____.

Results in a Business Calculus 1 Class: The Five Good Answers (of 36)

- The inverse of e^x
- The inverse of an exponential
- A function used to determine exponent
- The exponent required to produce a given #
- Inverse of Exponitioal [sic]

Results in a Business Calculus 1 Class: The Six Basic Answers (of 36)

- A function
- A function
- A type of function
- A function
- Function
- A mathmatic function [sic]

Results in a Business Calculus 1 Class: A Selection of the 25 Wrong Answers

- An expression to find unusual exponent rates
- A function that increases at a high rate
- Something I can use but can't define
- The derivative of an exponential
- Annoying
- The opposite of an exponent
- No idea but I think it has something to do with the number 10
- Base function (depending on specific base)
- One of the words for math. (I don't know)

Part 2 of the POP QUIZ!

Select ALL that apply:

A logarithm is:

a) A set of rules
b) An exponent
c) A number
d) An order of magnitude
e) A function
f) A transformation
g) An inverse

Some Part 2 Results

Answer	From 36 students in Business Calculus I	From 64 students in Calc 3 or Circuits (both with a Calc 2 prerequisite)
Function	69%	67%
Inverse	64%	65%
Number	58%	69%
Exponent	44%	71%
Set of Rules	50%	42%
Order of magnitude	42%	48%
Transformation	39%	41%

What are your thoughts about these numbers?

More Thoughts

- What issues do you find in teaching logs?
  - "Logs are just treated as another calculator exercise"
  - "Logs are just another section in a student's journey in math"
- What's in a log?

Understanding?

- Are we asking students to understand applications based on their [lack of] understanding of logs?
- Or are we asking students to understand logs based on their [lack of] understanding of applications?
- What applications can we expect students to understand?
  - Distance
  - Time
  - Money
  - Temperature
  - pH Levels
  - Earthquake Intensity

Distance

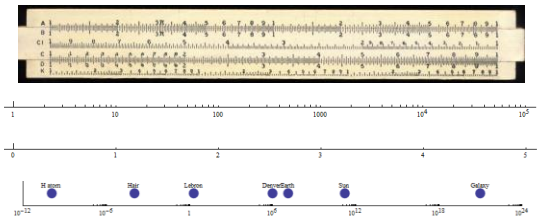
(how well do they – and we - really have a handle on it?)

- Collect the following information from the internet. Use the same unit of length for each. Do NOT use scientific notation.
  - The diameter of a hydrogen atom
  - Thickness of a human hair
  - The height of Lebron James (plays basketball for the Miami Heat)
  - The distance from Kansas City to Denver
  - The diameter of the earth
  - The distance from the earth to the sun
  - The diameter of the Milky Way Galaxy
- Compute the logarithm of each number.
- Explain how the logarithms are growing.
- Why use logarithms rather than the original number?

Results

Item	Values	Logarithms
Hydrogen atom	0.000000000106 m	- 9.80
Human hair	0.0001 m	- 4.00
Lebron James	2.01 m	0.30
Kansas City to Denver	970900 m	5.99
Diameter of the earth	12756000 m	7.11
Earth to Sun	150000000000 m	11.18
Milky Way Galaxy	95000000000000000000 m	20.98

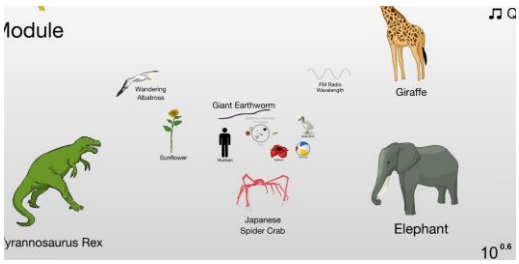
Log Scale on a Slide Rule



- How else can we simultaneously graph the very large and the very small?

Scale of the Universe

Flash object at http://htwins.net/scale2/

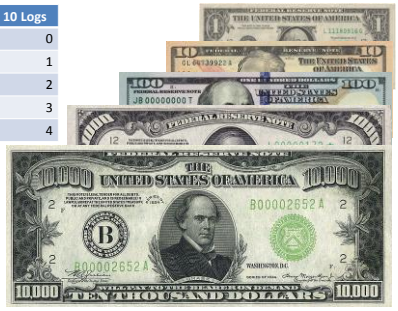


Open browser to run local copy

Money!

(they – and we - deal with this every day)

Money	Base 10 Logs
$1	0
$10	1
$100	2
$1000	3
$10000	4



Images from Wikipedia

Growth of Money

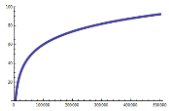
- At 5% interest compounded continuously, how long will it take for \$5000 to grow to \$*B*?

$$A = Pe^{rt}$$

$$B = 5000e^{0.05t}$$

$$t = 20\ln\frac{B}{5000}$$

B	t (years)
$5,000	0
$10,000	13.8
$15,000	21.9
$20,000	27.7
$40,000	41.0



- Which scale is the log scale?
  - Doubling Time (13.8 years) comes from a logarithmic expression.
  - Time is a logarithmic function of the account balance.
  - Time can describe the order of magnitude of the account balance.

One College Algebra Book's Presentation of Sound Intensity

- A definition: $B = 10\log\frac{I}{I_0}$ measures psychological sensation of loudness.
- The reference intensity: $I_0 = 10^{-12}\ \text{W/m}^2$
- Table of common sounds and their decibel levels

- Example: Find the decibel intensity level of a jet engine during takeoff if intensity was 100 W/m^2.

So what has the student learned? Psychology? Isn't this "just another calculator exercise"?

Sound Intensity: The Real Issue

- Circuit Gain
  - … or What Are you Doing to Your Ears?
- You increase the volume from 3.1 to 3.2. What is the percent increase in intensity on your ears?
  - Hint: 3% is wrong.
  - About a 26% increase!

$$\frac{10^{3.2}}{10^{3.1}} = 10^{0.1} \approx 1.2589$$

- What about an increase from 90.1 to 90.2 dB?

Noise!

(which they think they understand)

Table of sound levels L (loudness of noise) with corresponding sound pressure and sound intensity

Sound sources (noise) Examples with distance	Sound pressure Level L_p dB SPL	Sound pressure p N/m^2 = Pa Sound field quantity	Sound intensity I W/m^2 Sound energy quantity
Jet aircraft, 50 m away	140	200	100
Threshold of pain	130	63.2	10
Threshold of discomfort	120	20	1
Chainsaw, 1 m distance	110	6.3	0.1
Disco, 1 m from speaker	100	2	0.01
Diesel truck, 10 m away	90	0.63	0.001
Kerbside of busy road, 5 m	80	0.2	0.000 1
Vacuum cleaner, distance 1 m	70	0.063	0.000 01
Conversational speech, 1 m	60	0.02	0.000 001
Average home	50	0.006 3	0.000 000 1
Quiet library	40	0.002	0.000 000 01
Quiet bedroom at night	30	0.000 63	0.000 000 001
Background in TV studio	20	0.000 2	0.000 000 000 1
Rustling leaves in the distance	10	0.000 063	0.000 000 000 01
Hearing threshold	0	0.000 02	0.000 000 000 001

Table from: http://www.sengpielaudio.com/TableOfSoundPressureLevels.htm

Spent Fuel

(using understanding of logs for an unknown application)

- How long will spent fuel with a half-life of 87.7 years take to lose p of its radioactivity?

$$y = y_0 e^{-kt}$$

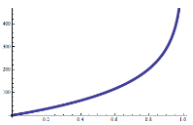
$$\frac{1}{2} y_0 = y_0 e^{-k(87.7)}$$

$$k = \frac{\ln 2}{87.7} \approx 0.0079$$

$$(1-p) y_0 = y_0 e^{-0.0079t}$$

$$t = -127 \ln(1-p)$$

p	t (yrs)
0.5	87.7
0.9	291.3
0.99	582.7
0.999	874.0
0.9999	1165.3
0.99999	1456.7



- Where is the log connection?
  - Time describes the order of magnitude of the remaining radioactivity.

What's in a Log?

- Functions, inverses, numbers
  - And about 2/3 of students are learning these
- Exponents
  - In spite of the emphasis from college algebra textbooks, it is taking a long time for students to learn this
- Order of Magnitude
  - Less than half of students know this, even though it describes a key feature of logarithms, and college algebra textbooks rarely mention it

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