



Spent Fuel and Circuit Gain: What's in a Log?

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Disclaimer

- This presentation is **NOT** about
 - doing logs
 - rules of logs
 - computing logs
 - calculations with logs
 - etc. and so forth...
- Rather, it is about
 - understanding logs conceptually ...

POP QUIZ !!!

- No discussions with your neighbor
- One minute time limit

- Complete the sentence:
A logarithm is _____.

**Results in a Business Calculus 1 Class:
The Five Good Answers (of 36)**

- The inverse of e^x
- The inverse of an exponential
- A function used to determine exponent
- The exponent required to produce a given #
- Inverse of Exponential [sic]

**Results in a Business Calculus 1 Class:
The Six Basic Answers (of 36)**

- A function
- A function
- A type of function
- A function
- Function
- A mathematic function [sic]

Results in a Business Calculus 1 Class: A Selection of the 25 Wrong Answers

- An expression to find unusual exponent rates
- A function that increases at a high rate
- Something I can use but can't define
- The derivative of an exponential
- Annoying
- The opposite of an exponent
- No idea but I think it has something to do with the number 10
- Base function (depending on specific base)
- One of the words for math. (I don't know)

Part 2 of the POP QUIZ!

Select ALL that apply:

A logarithm is:

- A set of rules
- An exponent
- A number
- An order of magnitude
- A function
- A transformation
- An inverse

Some Part 2 Results

Answer	From 36 students in Business Calculus I	From 64 students in Calc 3 or Circuits (both with a Calc 2 prerequisite)
Function	69%	67%
Inverse	64%	65%
Number	58%	69%
Exponent	44%	71%
Set of Rules	50%	42%
Order of magnitude	42%	48%
Transformation	39%	41%

What are your thoughts about these numbers?

More Thoughts

- What issues do you find in teaching logs?
 - “Logs are just treated as another calculator exercise”
 - “Logs are just another section in a student’s journey in math”
- What’s in a log?

Understanding?

- Are we asking students to understand applications based on their [lack of] understanding of logs?
- Or are we asking students to understand logs based on their [lack of] understanding of applications?
- What applications can we expect students to understand?
 - Distance
 - Time
 - Money
 - Temperature
 - pH Levels
 - Earthquake Intensity

Distance

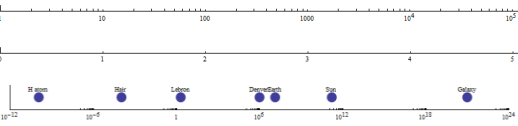
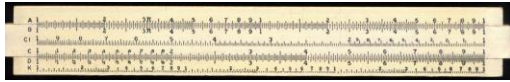
(how well do they – and we - really have a handle on it?)

- Collect the following information from the internet. Use the same unit of length for each. Do NOT use scientific notation.
 - The diameter of a hydrogen atom
 - Thickness of a human hair
 - The height of LeBron James (plays basketball for the Miami Heat)
 - The distance from Kansas City to Denver
 - The diameter of the earth
 - The distance from the earth to the sun
 - The diameter of the Milky Way Galaxy
- Compute the logarithm of each number.
- Explain how the logarithms are growing.
- Why use logarithms rather than the original number?

Results

Item	Values	Logarithms
Hydrogen atom	0.000000000106 m	- 9.80
Human hair	0.0001 m	- 4.00
Lebron James	2.01 m	0.30
Kansas City to Denver	970900 m	5.99
Diameter of the earth	12756000 m	7.11
Earth to Sun	150000000000 m	11.18
Milky Way Galaxy	95000000000000000000 m	20.98

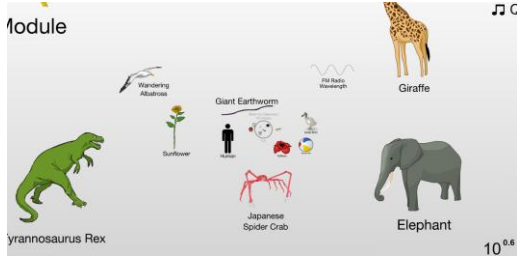
Log Scale on a Slide Rule



- How else can we simultaneously graph the very large and the very small?

Scale of the Universe

Flash object at <http://htwins.net/scale2/>

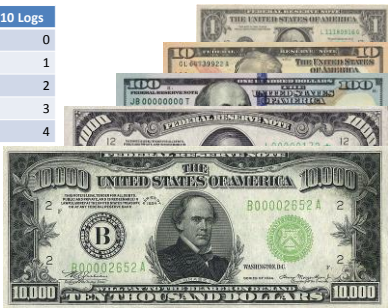


Open browser to run local copy

Money!

(they – and we - deal with this every day)

Money	Base 10 Logs
\$1	0
\$10	1
\$100	2
\$1000	3
\$10000	4



Images from Wikipedia

Growth of Money

- At 5% interest compounded continuously, how long will it take for \$5000 to grow to \$B?

$A = Pe^{rt}$
 $B = 5000e^{0.05t}$
 $t = 20 \ln \frac{B}{5000}$

B	t (years)
\$5,000	0
\$10,000	13.8
\$15,000	21.9
\$20,000	27.7
\$40,000	41.0

- Which scale is the log scale?
 - Doubling Time (13.8 years) comes from a logarithmic expression.
 - Time is a logarithmic function of the account balance.
 - Time can describe the order of magnitude of the account balance.

One College Algebra Book's Presentation of Sound Intensity

- A definition: $B = 10 \log \frac{I}{I_0}$ measures psychological sensation of loudness.
- The reference intensity: $I_0 = 10^{-12} \text{ W/m}^2$
- Table of common sounds and their decibel levels
- Example: Find the decibel intensity level of a jet engine during takeoff if intensity was 100 W/m^2 .

So what has the student learned? Psychology? Isn't this "just another calculator exercise"?

Sound Intensity: The Real Issue



- Circuit Gain
 - ... or What Are you Doing to Your Ears?
- You increase the volume from 3.1 to 3.2. What is the percent increase in intensity on your ears?
 - Hint: 3% is wrong. $\frac{10^{3.2}}{10^{3.1}} = 10^{0.1} \approx 1.2589$
 - About a 26% increase!
- What about an increase from 90.1 to 90.2 dB?

Noise!

(which they think they understand)

Table of sound levels L (loudness of noise) with corresponding sound pressure and sound intensity

Sound sources (noise) Examples with distance	Sound pressure Level L_p , dB SPL	Sound pressure p $N/m^2 = Pa$ Sound field quantity	Sound intensity I W/m^2 Sound energy quantity
Jet aircraft, 50 m away	140	200	100
Threshold of pain	130	63.2	10
Threshold of discomfort	120	20	1
Chinowse, 1 m distance	110	6.3	0.1
Disco, 1 m from speaker	100	2	0.01
Diesel truck, 10 m away	90	0.63	0.001
Karbside of busy road, 5 m	80	0.2	0.0001
Vacuum cleaner, distance 1 m	70	0.063	0.00001
Conversational speech, 1 m	60	0.02	0.000001
Average home	50	0.0063	0.0000001
Quiet library	40	0.002	0.00000001
Quiet bedroom at night	30	0.00063	0.000000001
Background in TV studio	20	0.0002	0.0000000001
Rustling leaves in the distance	10	0.000063	0.00000000001
Hearing threshold	0	0.00002	0.000000000001

Table from: <http://www.sengpielaudio.com/TableOfSoundPressureLevels.htm>

Spent Fuel

(using understanding of logs for an unknown application)

- How long will spent fuel with a half-life of 87.7 years take to lose p of its radioactivity?

$$y = y_0 e^{-kt}$$

$$\frac{1}{2} y_0 = y_0 e^{-k(87.7)}$$

$$k = \frac{\ln 2}{87.7} \approx 0.0079$$

$$(1-p)y_0 = y_0 e^{-0.0079t}$$

$$t = -127 \ln(1-p)$$

p	t (yrs)
0.5	87.7
0.9	291.3
0.99	582.7
0.999	874.0
0.9999	1165.3
0.99999	1456.7

- Where is the log connection?
 - Time describes the order of magnitude of the remaining radioactivity.

What's in a Log?

- Functions, inverses, numbers
 - And about 2/3 of students are learning these
- Exponents
 - In spite of the emphasis from college algebra textbooks, it is taking a long time for students to learn this
- Order of Magnitude
 - Less than half of students know this, even though it describes a key feature of logarithms, and college algebra textbooks rarely mention it

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